

A MATHEMATICAL RELATIONSHIP BETWEEN GROWTH AND SCALING IN AN EXPONENTIALLY GROWING BIBLIOGRAPHIC SYSTEM



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Abstract: We establish a mathematical relationship between growth and scaling in an exponentially growing bibliographic system with constant growth in citations per normalized number of publications.

Keywords: scaling; exponential growth

1. INTRODUCTION

In bibliometric evaluation, contractees frequently request the quantification of the growth of indicators. That is, they want to

know the growth of certain objects of study over a time span in one number. Sometimes (see e.g. Schmoch et al., 2006), the request for the growth of the number of publications is met by providing the Sharpe Ratio

– a kind of weighted, average, normalized, annual growth rate (Sharpe, 1994).

Recently Lietz and Riechert (2013) have shown that the Sharpe Ratio can also be applied to bivariate indicators such as productivity (number of publications per author) and discussed scaling exponents (Bettencourt et al., 2008) as an alternative to this approach. Scaling analysis can be considered a modeling technique but it can also be used for evaluation purposes. However, it is only applicable to bivariate data. In the case of publications and authors, the scaling exponent is a system-specific descriptor of how productivity changes as the number of authors of an object of study varies over time.

Calculating the Sharpe Ratio and performing scaling analysis are two different perspectives on growth, the former relying on simple parametric statistics, the latter studying systems' not necessarily linear evolutionary dynamics. Recall that the Sharpe Ratio is a normalized and weighted indicator, properties the exponent does not have. Nevertheless, studying the productivity (number of journal articles per author) in 27 scientific disciplines using the *Scopus* database in time windows of up to 13 years, the Pearson correlation of the Sharpe Ratios and scaling exponents was shown to be at least 0.67 (Lietz & Riechert, 2013).

This correlation remains to be explored mathematically. To do so, we relate the simple average annual growth rate of a system to its scaling exponent. Further assuming that growth is constant, our question is: What is the scaling exponent if this constant growth rate is given? Instead of productivity, we use the notation of impact (citations per publication) because CpP is more established as an indicator, but the basic idea is the same. Scaling analysis of impact was pioneered by Katz (1999), be it in a structural, not dynamical sense. Recent applications related to scaling include (Gao & Guan, 2009; Gao et al., 2010).

2. NOTATIONS

Let $p(t)$ be the size of a publication corpus at time t and $p(0) > 0$. Assuming exponential growth we have

$$p(t) = p(0)e^{at} \quad (1)$$

where we assume that $a > 0$ is a constant. We normalize the number of publications by introducing

$$q(t) = \frac{p(t)}{p(0)} \quad (2)$$

leading to

$$q(t) = e^{at}. \quad (3)$$

In this way $q(0) = 1$.

The total number of citations received by this corpus at time t is denoted by $c(t)$. Normalizing in a similar way we introduce

$$d(t) = \frac{c(t)}{c(0)}. \quad (4)$$

Next, we come to our research question by making two more assumptions:

1) $d(t)$ and $q(t)$ scale as

$$d(t) = q(t)^\beta \quad (5)$$

where β is the scaling exponent.

2) d/q has a constant growth rate $g > 0$, expressed as

$$d(t) = q(t)(1+g)^t. \quad (6)$$

3. RELATIONSHIP BETWEEN GROWTH AND SCALING

Our research question stated in the introduction translates into: What is the relationship between the constants g and β ?

From equation (3) we obtain

$$t = \frac{\ln(q(t))}{a}. \quad (7)$$

Putting equation (3) into (6) we get

$$d(t) = e^{at}(1+g)^t = (e^a(1+g))^t \quad (8)$$

or

$$t = \frac{\ln(d(t))}{\ln(e^a(1+g))}. \quad (9)$$

Setting (7) equal to (9) we get

$$\frac{\ln(q(t))}{a} = \frac{\ln(d(t))}{\ln(e^a(1+g))} \quad (10)$$

or

$$\ln(d(t)) = \frac{\ln(e^a(1+g))}{a} \ln(q(t)). \quad (11)$$

Taking the natural exponential of (11) gives

$$d(t) = q(t)^{\frac{\ln(e^a(1+g))}{a}}. \quad (12)$$

Finally equations (5) and (12) lead to the desired relation:

$$\beta = \frac{\ln(e^a(1+g))}{a} + \frac{\ln(1+g)}{a}. \quad (13)$$

In words, in an exponentially growing publication corpus with a constant impact growth g the scaling exponent β describing the change in the normalized citations per publication is a function of g . Equation (13) provides the exact relation. For example: if $a = 1$ and $g = 0.05$, then $\beta = 1.05$; if $a = 1.2$ and $g = 0.1$, then $\beta = 1.08$.

4. CONCLUSION

We derived a theoretical formula for the scaling exponent in an exponentially growing bibliographic system with constant growth in citations per publications (CpP).

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